METHODS ON LOCAL HIT RECONSTRUCTION IN THE PIXEL DETECTOR FOR THE BELLE II EXPERIMENT

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Diplomarbeit

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Yet Nature is made better by no mean.
But Nature makes that mean:
So, o’er that art,
Which you say adds to Nature,
Is an art,
That Nature makes.

William Shakespeare
The Winter’s Tale
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Chapter 1

Introduction

For a long period of time, the laws of physics had been believed to be invariant for any physical process under the transformation of spatial mirror inversion. In other words, there was always thought to be a “parity symmetry” dominating all the physical interactions in the Universe. In particle physics, this law of parity conservation had naturally been adapted to all sorts of particle interactions that take place through the electromagnetic force, strong nuclear force and the weak nuclear force, which are alone the three types of fundamental interactions concerned in the study of particle physics.

In 1956, the parity conservation under the weak interaction was first called in question by two theoretical physicists [1]. Soon an experiment designed for investigation of the weak beta decay of Cobalt-60 nuclei demonstrated a clear discrepancy on the reaction rates of the decay processes and the ones undergoing in their mirror image [2]. This shocking phenomenon lead to the conclusion that the parity symmetry is indeed violated under certain weak interactions. Thus, the parity violation was discovered as bringing out a great surprise.

By applying a mathematical principle of the Hilbert space, a new symmetry which is a combined symmetry of the charge conjugation (“C”) and the parity transformation (“P”) was proposed by Lev Landau in 1957. Similarly, charge conjugation was also believed to be invariant under all physical interactions, which means, the laws of physics shall remain the same after changing the matter to their own antimatter that go through the same physical process. The “CP symmetry” was then considered to be able to restore the symmetry violation of parity while at the same time revealing the true symmetry between matter and antimatter.

Unexpectedly, in 1964 an experiment conducted on investigating the weak decay of the neutral kaons disproved this postulate [3]. The results showed that neither the
charge conjugation (“C”) or its combination with parity (“CP”) had been conserved during the decay of the neutral kaons. The reaction rates for the processes with the neutral kaons turning into their antiparticles and vice versa were slightly different. This striking phenomenon observed again under the weak interaction clearly confirmed the violation of the proposed “CP symmetry”.

CP violation was first revealed as being inconsistent with the Standard Model of that time. In 1973, M. Kobayashi and T. Maskawa made a proposal that the quark mixing could actually be the origin of CP asymmetry [4]. Their theory pointed out there should be at least three generations of quarks in Nature, which means the Cabbibo matrix describing the flavor-changing behavior for only two generations of quarks should be modified to a $3 \times 3$ matrix. Thus the Cabbibo-Kobayashi-Maskawa matrix (CKM matrix) was proposed, bringing totally four independent parameters that are physically significant instead of just one from the former Cabbibo matrix. Among these four variables, three are real and represent the mixing angles for the flavor transitions between any two of the postulated three quark generations, while the other one which is a complex phase taken to be the source of CP violation. This “CP phase” made the Standard Model again compatible with observations and the hypothesis of three quark generations required by the KM theory also pointed out the direction for the new experiments.

The third generation quarks, bottom ($b$) and top ($t$) were separately discovered at Fermilab and Tevatron years later [5] [6], partially confirming the KM theory. To fully prove that the quark mixing is indeed the real source of CP violation, the B-factories were set up for further investigation. The general concept is to produce large number of B mesons containing the bottom quarks and their antiquarks ($b, \bar{b}$), which is the only third generation quark flavor that can be found in a bound state. Through measuring various decay modes of the B mesons and their antiparticles, CP violation can be observed within certain types of decay processes. In 2001, the BaBar Experiment at the Stanford Linear Accelerator Center (SLAC) and the Belle Experiment at the High Energy Accelerator Research Organisation (KEK) both observed and made precise measurements on the time-dependent CP violation process $B^0(\bar{B}^0) \rightarrow J/\psi K^0_S$ [7] [8]. With great significance, CP violation outside of the kaon system was observed for the first time, and as anticipated, the resulting data of the measurements confirmed that the quark-mixing is truly the source of the observed CP asymmetry in Nature. Thus, the KM theory was verified in explaining the phenomenon of CP violation. In 2008, Kobayashi and Maskawa were awarded the Nobel Prize in Physics for their discovery of the origin of the broken CP symmetry which also indicated the existence of at least three quark generations [9].
Since 2001, the B-factories have made further discoveries through the observation on B-decays. Among them the Belle Experiment conducted at the KEKB collider in Japan has made many achievements, such as precise measurements of the CKM matrix elements [10], discovery of direct CP violation in $B^0 \rightarrow \pi^+\pi^-$ [11] and $B^0 \rightarrow K^+\pi^-$ [12], and the observation of rare B-decay processes of $B \rightarrow \tau\nu$ [13] and $B \rightarrow \rho\gamma$ [14]. All these exciting results originate from the electron-positron ($e^-e^+$) collisions with a center-of-mass energy set to Υ(4S) resonance, which only slightly exceeds the mass of two B mesons ($B^0, B^\pm$), leads to the creation of $B\bar{B}$ pairs without producing associating particles which would cause extra background. For measurements of the critical parameters of CP violation, the beam energy is set to be asymmetric, so that the produced B mesons are always boosted in one direction obeying the conservation of momentum. Since the velocity of the B mesons can be obtained from calculation, through precise location reconstruction of the decay vertices (vertexing), the different decay time of B mesons and their antiparticles can be accurately measured, which is the key element in the study of CP violation.

More experiments focusing on CP violation and B-activities are still being conducted or in preparation, aiming towards deeper understandings in numerous yet unsolved fundamental problems in modern physics. For instance, while revealing the asymmetry of the laws of physics governing the matter and antimatter, CP violation has been widely adapted to form the theories which intend to explain the vast discrepancy of the matter and antimatter in the Universe. According to the observations of the cosmological radiation, it is widely accepted, that the Big Bang produced greatly more matter, which formed our familiar surroundings, than the antimatter. The exact reason for this imbalance is yet unknown, since the current knowledge on CP violation is unable to fully explain the magnitude of the matter-antimatter asymmetry. Therefore, the hypothesized “out of Standard Model” CP violation which does not originate from the complex phase of the CKM matrix, is of great interest for the next generation B-experiments.

Since the BaBar Experiment and the Belle Experiment have both completed data collection in 2008 and 2010, a new set of B-factories have later been proposed. Among these the LHCB Experiment at the Large Hadron Collider (LHC) in Geneva, Switzerland, is already in operation. Meanwhile, Belle II, which is an upgrade of the previous Belle Experiment at KEK, is currently under preparation. One of its remarkable improvements is, that the luminosity of the SuperKEKB accelerator, on which the Belle II Experiment will be conducted, will be increased to about 40 times to its previous Belle level, as shown in Figure 1.1. Achieving an instantaneous luminosity of $8 \times 10^{35} cm^{-2}s^{-1}$, the SuperKEKB collider will maintain the world record of possessing the highest luminosity made by its predecessor. Consequently, Belle II will produce a much larger number of B
mesons, and as a result, it will significantly increase the detector sensitivity for exploring the widely anticipated New Physics phenomena.

Besides the upgrade made on the KEKB accelerator, Belle II also adopted a greatly modified detector design, including the enhancements of the vertex detectors through adding the Pixel Detector (PXD) and the improvements made on the Silicon Vertex Detector (SVD), aiming an optimal reconstruction resolution on locating the decay vertices of B mesons, which is the key essence of the measurements on CP violation.

Since the positions of the decay vertices are estimated through *vertexing*, as illustrated in Figure 1.2, a process that evaluates the track parameters of the charged decay products through analyzing the best possible hit positions on the vertex detectors. Therefore, the hit position resolution for measurements conducted on the vertex detectors, namely the SVD and the PXD, is of particular interest for a thorough study of CP violation. Thus, while the detector hardware is largely upgraded, the reconstruction software responsible of locating the hit positions on the vertex detectors is also being built up for an enhanced vertexing precision. More specifically, new position finding algorithms for the Pixel Detector that could lead to a higher resolution on estimating the positions of the decay vertices are being intensively tested. In Chapter 4, the “Shift Algorithm” which
could replace the currently integrated Center of Gravity (COG) algorithm for the PXD hit position estimation will be introduced. It will be shown, that this new algorithm offers a considerably sharper resolution in reconstructing the local hit positions with the simulated PXD cluster data. Furthermore, the ways of testing this new algorithm with the more realistic basf2 simulation data and the future integration of this algorithm into the basf2 software framework will be briefly discussed.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1_2.png}
\caption{The reconstructed hit positions are obtained through analyzing the data measured by vertex detectors. Vertexing provides the estimated track parameters and hence the approximate vertex position.}
\end{figure}
Chapter 2

The Belle II Detector

In this chapter the Belle II detector will be introduced with an emphasis on the vertex detectors (PXD, SVD). Other detector components will be shortly described. The detailed information can be looked up from the “Belle II Technical Design Report” [16].

2.1 General View

Despite of the intention for a deeper understanding of the CP violation, Belle II also targets the discovery of other potential New Physics phenomena. The Belle Experiment has already observed various rare, possibly “out of the Standard Model” processes. But due to the lower luminosity of the previous KEKB accelerator, many observations were statistically limited to be confirmed as New Physics. For the Belle II Experiment, the upgraded accelerator – SuperKEKB, which would reach an instantaneous luminosity of $8 \times 10^{35} cm^{-2}s^{-1}$, is expected to answer a set of “New Physics related” questions. Its integrated luminosity is about 50 times greater than the previous one for the Belle Experiment, which indicates a roughly 50 times increase on the physics event rate. Therefore, Belle II will achieve a significantly higher sensitivity for many observables which could lead to determinations of whether many rarely observed processes were indeed New Physics effects.

Such high luminosity of the SuperKEKB accelerator is to be realized by applying a smaller beam size under the “Nano beam” scheme and a doubled beam current. The asymmetric beam energies are modified to $4.0 GeV$ and $7.0 GeV$, respectively. This lower asymmetry sacrifices the physics sensitivity but is essential for suppressing the Toushek effect in order to achieve the desired luminosity.
Besides the upgrade on the KEKB accelerator, the Belle II detector components have also been re-engineered to bring various improvements and to cope with the foreseeable problems brought by higher luminosity. As in Belle, these components are installed centering around the Interaction Point, which is the collision point situated in the beampipe. A cross-section of the overall Belle II detector geometry is shown in Figure x.x.

The innermost vertex detector, which is just right outside the beampipe, is the 2-layer Pixel Detector (PXD). Providing more channels from the pixel sensors and hence possessing less occupancy of each channel, compared to the former Silicon Strip Detector, the PXD is completely capable of collecting data for highly accurate vertex reconstruction in the largely increased luminosity environment, while being positioned at a very close range to the Interaction Point. In addition, 4 layers of expanded Silicon Vertex Detector (SVD) are placed outside of the PXD, which are also vital for precise vertex reconstruction. To more accurately measure the tracks’ position and momenta of the charged particles, the Central Drift Chamber (CDC) enclosing the vertex detector, is modified to have a larger radius and smaller drift cells. Further out, completely new particle identification components with much faster read-out electronics, the Cherenkov imaging detectors – TOP and ARICH, respectively, are placed in the barrel and end-cap region. Surrounding them is the electromagnetic calorimeter (ECL), which is used for detection and measurements of the photons as well as electrons. At the outermost region of the detector, is the superconducting magnet generating the magnetic field for CDC, enclosed by the KLM detector, which identifies the neutral K-long mesons and the muons.

More information for these components will be given in the following sections.

### 2.2 Vertex Detector

#### 2.2.1 Pixel Detector (PXD)

For the Belle II Experiment, the SuperKEKB luminosity generates severe background environment, including the beam-related background and low-momentum-transfer QED processes. And under the nano-beam scheme, the radius of the beampipe around the Interaction Point is reduced to 10\(\text{mm}\), which is in principal ideal for vertex reconstruction, but meanwhile enlarges the background effect by producing a very close range between the vertex detector and the Interaction Point. Therefore, the innermost layers of the vertex detector should be re-designed to maintain precise vertex reconstruction under such challenging environment. To do so, the Silicon Strip Detector of the previous Belle...
Experiment is replaced by a detector (PXD) made of Pixel Sensors, which provide much more channels to separate the intense background signals and the event signals.

The Pixel Detector has two layers of overall 8 million pixel sensors positioned in the area directly outside the beampipe, as shown in Figure 2.1.

The inner and outer radius of these layers are currently chosen to be 14\(\text{mm}\) and 22\(\text{mm}\). The overall size of the Pixel Detector is rather insignificant, but it is the most vital component for accurately locating the decay vertices of various physical processes of interest, which is the key to precise measurements of B-decay times.

The concept of setting up the pixel detector followed by strip detectors is adapted from the LHC Experiment. But Belle II has a much lower energy compared to the LHC experiment and hence the “regular” silicon sensors that are relatively thick would cause too much multiple scattering to achieve the desired tracking resolution for Belle II. Therefore the pixel sensors have to be thinned down to reduce its substrate that contributes to multiple scattering. However, the thinning also diminishes the Signal-to-Noise ratio and hence would eventually reduce the position finding resolution. As a result, a more advanced type of silicon pixel sensors, i.e. the DEPleted Field Effect Transistors (DEPFET) was adapted by the Belle II PXD. The DEPFET sensors are thinned down to only 75 microns and meanwhile are still able to provide an optimal \(S/N\) ratio. In addition, they consume very little energy and thus can be cooled down.
simply by air. Thus the readout electronics and its affiliating cooling system can be placed outside of the acceptance region, effectively reducing the amount of material contributed to extra multiple scattering.

The DEPFET sensor (as shown in Figure 2.2) is a semiconductor detector invented in 1987. It contains a p-channel MOSFET or JFET which is integrated on a silicon detector substrate. When a high negative voltage applied to its p+ contact, the substrate becomes fully depleted. About 1µm under its transistor channel, a potential minimum called the “internal gate” is implanted through sideward depletion and doping. The incident particles traveling through the fully depleted bulk create electron-hole pairs. Naturally the positively charged holes drift to the back contact, while the electrons are collected at the internal gate. Once the sensor is switched on, the collected electrons modulate the p-channel current. By simply measuring this current, the charge created by incident particles can be “non-destructively” read out, which means this process can be repeated many times without triggering any variation on the values. To remove the collected signal charges at the internal gate, a “Clear” process takes place by applying a positive voltage pulse on the n+ contact.

Due to leak currents between the silicon detectors and the effects of radiation, there will always be Gaussian formed electronic noise measured in the pixel sensors, even when there’s no event charge generated by any incident particles. This effect does reduce the reconstruction resolution on certain level, and will also be taken into account for the PXD simulation.
2.2.2 Silicon Vertex Detector (SVD)

The Silicon Vertex Detector is an important component of the Belle II vertex detector. Installed between the Pixel Detector and the Central Drift Chamber, it provides data to extrapolate the tracks reconstructed in the CDC back into the PXD with high efficiency. Through the extrapolation, the “correct” PXD clusters which are not lit on due to background or radiation effects will be analyzed for the determination of the exact track parameters that are required for precise vertexing. Additionally, the SVD also plays an important role on measuring the vertex information of low-momentum particles which mostly cannot propagate further enough to the CDC, like the $D^*$ daughters, which can be used to determine the flavor the parent B meson.

The SVD in Belle II has been improved for the quality and efficiency on charged particles’ reconstruction compared to Belle’s SVD2. It is made of four layers of double-sided silicon strip detectors (DSSDs), which are 6-inch wafers that are currently the largest DSSDs can be made. Thus the cost and channel counts are effectively reduced. Each double-sided DSSD sensor has the “p-side” with long p-doted strips parallel to the direction of the beam axis, while the other side - “n-side” has the short n-doted strips perpendicular to the direction of the long p-doted strips. The p-side of every DSSD is arranged to face towards the beampipe. When a central magnetic field parallel to the beam axis is applied, the electron-hole pairs created by any incident particle traveling through a DSSD would be separated. The electrons would be then collected on a short strip on the “n-side” while the holes being collected on a long strip of the “p-side”. Thus, the intersection point of the particle’s track with this DSSD wafer on any particular SVD layer can be located with the indices of the corresponding p-/n-strips.

Concerning the radii of the PXD and the CDC, the inner radius of the SVD has been chosen 38 $mm$ and the outer radius 140 $mm$. The SVD also covers the polar angular acceptance of the Belle II detector with a range from 17 degree to 150 degree. As shown in Figure 2.3, this angular range is asymmetric, which is a consequence of the forward boost of the B mesons created by the asymmetric energy beams. Compared to the SVD2 of the Belle Experiment, the radial coverage is almost doubled, hence more DSSD wafers would be required under the previous cylindrical geometry. Therefore, the wafers in the forward region are designed to be slanted, leading to a lantern-shaped geometry. In order to avoid any overlap among them, the slanted detectors have to be made to one standard trapezoidal shape, and carefully arranged to certain slant angles that differ from one layer to another.
Interestingly, another small but important adjustment on the sensors’ placements should be made due to the higher mobility of the electrons compared to their pairing holes.

As in Figure 2.4 (left) demonstrated, in the usual configuration the electrons have a much larger deflection than the “heavier” holes. The relatively large area accepting the widely deflected electrons on the n-strip would induce image currents in a few p-strips on the other side of DSSD wafer. This surely would affect the determination of the track’s position. Therefore, the electron spread on the n-strip should be minimized by tilting all the detectors in a certain angle, as in Figure 2.4 (right). Consequently, the tilting leads to a “windmill structure” on the SVD geometry, illustrated in Figure 2.5.

The Silicon Vertex Detector of Belle II also has a low-mass design and is mechanically stable. It is supposed to work efficiently in providing precise reconstruction data under the SuperKEKB high luminosity environment.
2.3 Central Drift Chamber (CDC)

The Belle II Central Drift Chamber adapted the global structure of its predecessor of the Belle Experiment. Due to the larger SVD, its inner radius is increased to 160 mm while the outer radius increased to 1130 mm, resulting a much larger chamber volume integrated with an upgraded wire configuration (see Figure 2.6) for more accurate and efficient measurements.

Containing 14,336 sense wires and the chamber volume filled with helium-ethane gas, the CDC collects data triggered by the ionization process of the charged particles passing through the gas chamber, along with the advanced tracking algorithm, it’s designed to perform precise reconstruction and momenta measurements of the charged tracks. The
CDC also serves to determine the identities of various charged particles by measuring their energy loss within its gas volume.

The reconstructed tracks in the CDC provide vital information for the PXD hit reconstruction as well. These tracks can be linked back to the PXD clusters via extrapolation with help of the SVD, and hence help to exclude many PXD background clusters, while providing the direction of the charged tracks which create the event clusters. This piece of information is essential when applying the new algorithm on estimating the intersecting position of a reconstructed charged track with a particular PXD layer (see Chapter 4).

2.4 Particle Identification System (PID)

2.4.1 Time-of-Propagation (TOP) Counter

The PID located in the barrel region right outside of the CDC wall is the Time-Of-Propagation (TOP) counter. It is a uniform and compact replacement of Belle’s time-of-flight and aerogel Cherenkov counters, designed to provide a better efficiency on separating $K$ and $\pi$. One radiator module of the TOP counter is illustrated in Figure 2.7.
The incident particles propagate through the quartz radiator with a velocity greater than the speed of light, hence triggering the Cherenkov effect emitting photons. The Cherenkov photons would be internally reflected inside the radiator until reaching the photomultipliers (PMTs) at the end surfaces of the quartz bar, where their time of propagation are precisely measured. Under a “focusing scheme” the TOP also measures the impact position of the Cherenkov photons as 2-dimensional information $(x, y)$. Along with the precise timing, the Cherenkov ring image can be reconstructed, leading to the calculation of the Cherenkov angles which could reveal the identities of the corresponding incident particles.

### 2.4.2 Aerogel Ring-Imaging Cherenkov (ARICH) Detector

The Aerogel Ring-Imaging Cherenkov detector is situated in the forward endcap region. It is designed to provide good separation between kaons and pions over most of their momentum spectrum, while distinguishing slow pion, muon and electron with momentum under $1\text{GeV}$.  

As shown in Figure 2.8, the ARICH consists of the aerogel layers as the radiator. The Cherenkov photons propagate through the expansion volume towards the photon detector, which is the PMTs that are able to separate the Cherenkov rings formed by the photons. To achieve the required resolution for the measurements of the Cherenkov angles, the thickness of the aerogel radiator is restricted to $2\text{cm}$ and the expansion gap limited to $20\text{cm}$.  

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**Figure 2.7:** An illustration of TOP counter. Adapted from [16] p.220.
2.5 Electromagnetic Calorimeter (ECL)

The electromagnetic calorimeter is positioned directly outside the Belle II PID. Weighing 43 tons and consisting of 8736 scintillating CsI(Tl) crystals, it is mainly designed to precisely measure the photon energy and position, identifying electrons, detecting K long together with the KLM, and generating trigger signals for other detectors.

2.6 K-long and Muon (KLM) Detector

The K-long and muon detector is installed according to a sandwich structure. It consists of 4.7 cm thick iron plates enclosing the ECL, serving as the flux return for the superconducting solenoid while providing the interaction region where the neutral K-long mesons’ hadronical showering takes place. The active detector elements which are the Resistive Plate Chambers (RPCs) are located outside of the solenoid, providing effective identification of the penetrating muons. And in combination with the ECL, the potential neutral K-long candidates can also be determined.
Chapter 3

Physics Process and Toy Simulation of the Pixel Detector

The Pixel Detector (PXD) is the key component of the vertex detector designed to meet the Belle II physics requirements for precise vertex reconstruction. Working along with the Silicon Vertex Detector (SVD) and the Central Drift Chamber (CDC), it is expected to deliver excellent spatial resolution for the track impact parameters. Its anticipated superior performance will help to provide significantly improved resolution of the reconstructed decay vertices, which is essential for a thorough investigation on CP violation and other proposed New Physics effects.

This chapter will first provide an overview on the physics process underlying the PXD data collection and the expected energy loss distribution in the pixel sensors. In the first section, the process of ionization and the working principles of the silicon detector will be introduced, with an emphasis on the effects of the high-energy-transfer incidences and $\delta$ electrons. The second section is dedicated to the Toy Simulation which is made to generate the pixel cluster data for single track events. The simulation adapts the energy loss distribution addressed at the beginning of this chapter, while taking account of the Gaussian distributed electronic noise caused mainly by the leakage current. In the next chapter, data produced by this simulation will be used for the first stage testing of the new PXD local reconstruction algorithm.
3.1 The PXD Physics Process

3.1.1 Ionization

The DEPFET pixel sensor which constitutes the PXD is a typical semiconductor detector. It measures the signal current induced by the traversing low-energy charged particles which interact with its detector material through ionization.

The ionization process underlies the working principle for a large family of particle detectors, most commonly the gas detectors and the solid state detectors that are widely used for particle tracking and vertexing. The charged particles which propagate through these detectors interact with the atomic electrons of the detector material through electromagnetic interaction, transferring parts of their energy to these “scattered” electrons, subsequently leading to excitation or ionization of the corresponding atoms. The excited atoms would spontaneously return to their ground energy states by emitting photons without losing any of their bounded electrons. But when sufficient energy was transferred during the interaction, the electron which absorbed the proper amount of energy would escape from the atomic bound state and become freely to move, hence the atom is ionized.

Naturally the ionizing incident particles lose certain amount of their energy while propagating through the matter. The energy loss strongly depends on the particle’s momentum. Within certain momentum range, the average energy loss can be described by the Bethe-Bloch formula [17]:

\[ -\langle \frac{dE}{dx} \rangle = K z^2 Z \frac{1}{A} \frac{Z}{\beta^2} \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\text{max}}}{I} - \beta^2 - \frac{\delta(\beta \gamma)}{2}, \]

In this equation, $\beta \gamma$ is proportional to the incident particle’s momentum, $z$ represents the particle’s charge, $Z$ for the atomic number of the traversed material, $I$ for the mean excitation energy, $A$ for the atomic mass, $T_{\text{max}}$ for the maximum energy transfer of one collision, and $K$ short for $4\pi N_A r^2 m_e c^2$. For muons propagating through copper, Figure 3.1 demonstrates the relationship between the energy loss ($\frac{dE}{dx}$) and momentum ($\beta \gamma$) according to this equation. As shown in this figure, the muons with a momentum at around $\beta \gamma = 4$ possess the smallest cross section with the atomic electrons in copper. Thus in this momentum region the minimal ionization is reached as the muons’ energy loss is minimized. Particles like this are usually referred as minimum-ionizing particles (MIPs).
3.1.2 PXD as a Silicon Detector

As mentioned in the last section, the DEPleted Field Effect Transistor (DEPFET) made from semiconductor material, namely silicon, is the elementary component constituting the Pixel Detector. Since the 1980s, the Silicon Detector has been developed and widely applied for precise particle tracking and vertexing of various High Energy Particle experiments.

Compared to the gas tracking detector, the semiconductor detector possesses several considerable advantages. As a solid state detector, the higher density of the semiconductor material leads to much higher energy loss within a short distance, hence the free charge carriers generated from the ionization are created within a few microns of the path of the incident particle. Thus the diffusion effect is much less significant than in the gas detector, achieving an accurate position resolution of normally less than 10µm. In addition, the ionization energy needed to create free charges in the semiconductor is much less than in the gas chamber. Usually only a few electron-volts is required for a successful ionization process to take place in the semiconductor, while 20 − 40eV needed in the gas detector. This clearly indicates that the semiconductor detector provides...
a much larger Signal to Noise Ratio for the detection and measurements of charged particles.

As in Figure 3.2 illustrated, the semiconductor has a relatively narrow energy gap between the conduction energy band and the valence energy band. Thus, unlike the isolator, at room temperature some electrons in semiconductor already spontaneously occupy the conduction band and become freely to move. Therefore, a low intrinsic conductivity exists within the semiconductor and is caused by the thermal charge carriers, namely the electron-hole pairs. For pure silicon which has an energy gap of only 1.12eV, the required ionization energy to create one $e^-h^+$ pair is 3.62eV, which is optimal for producing good signal current. But its intrinsic free charge carriers, the thermal $e^-h^+$ pairs, could lead to a very strong noise signal during the read-out. For measurements on the MIPs, this so-called leakage current caused by the intrinsic $e^-h^+$ pairs in silicon, could be as 4 orders of magnitude larger than the signal current induced by the incident particles. Thus the intrinsic charge carriers have to be removed to reduce the leakage current in order to provide a low-noise environment.

![Graphical illustration of the band structure for isolator, semiconductor and metal.](image)

**Figure 3.2:** Graphical illustration of the band structure for isolator, semiconductor and metal. Taken from [18] p.7.

The depletion zone with virtually no intrinsic $e^-h^+$ pairs can be created by forming reverse biased p-n junctions. The silicon doped with element 5 atoms such as phosphorus atoms as donors receive extra conduction electrons and a raised Fermi energy level $E_F$, namely the n-doping. On the other hand, the p-doping in silicon creates extra free holes by adding element 3 atoms like boron atoms as acceptors, at the mean time causing a lowered level of Fermi energy. A p-n junction is created by attaching the n-/p-doped silicon, at their interface the diffusion occurs due to the difference of Fermi energy levels of both sides, meaning the extra free electrons and holes move toward the other side and recombine with each other, forming an expanding region free of charge carriers. The
once neutral n-/p-doped sides become charged during the diffusion and create an electric field which eventually stops the diffusion process, hence leaving a stable space charge region under the thermal equilibrium, namely the depletion zone.

When an external voltage is applied with the cathode to the p-doped side and the anode to the n-doped side, also known as the reverse bias, as shown in Figure 3.3, more electrons and holes from the bulk are pulled into the p-n junction and recombine with each other, therefore the size of the depletion zone is increased.

![p-n junction with reverse bias](image)

**Figure 3.3:** Schematic view of a p-n junction with reverse bias. Taken from [18] p.17.

This enlarged depletion zone is the way of constructing silicon particle detectors. At room temperature, there is only very small amount of thermally generated $e^{-}h$ pairs in the silicon detector material which is evidently a depletion zone. Consequently, as the reverse bias voltage is applied, the few $e^{-}h$ pairs are separated by the electric field and drift toward the electrodes causing the leakage current. This is the main source of the noise signal during read-out. Such electronic noise brings a slight background charge deposit which is approximately Gaussian distributed. This type of background charge will be taken into account during the Toy Simulation.

The silicon strip detector, as illustrated in Figure 3.4, is the most common type of silicon detector used for precise position measurements of the charged particles. Between the p-doped and n-doped narrow silicon strips which are used as electrodes, the pure silicon bulk is slightly n-doped and fully depleted of free charges, serving as the detector material. Within this region the traversing incident particles create $e^{-}h$ pairs through ionization. The $e^{-}h$ pairs which arise at very close distance to the particle’s trajectory would then drift to the electrodes, hence generate the signal current, which will be amplified for read-out. Through the signals of the individual strips, the track position of the incident particle is then determined.
Compared to the silicon strip detectors, the pixel detector supplies comprehensive 2D information for the particle tracking and provides higher resolution on estimating the position of incident particles. The pixel size of the PXD’s DEPFET sensor is thinned down to only $75\mu m$, therefore delivering an excellent spatial resolution, while consuming very low power and minimizing the effect of multiple scattering. Thus the pixel detectors are widely adapted as the innermost vertex detector for the upcoming High Energy Physics experiments.

### 3.1.3 Energy Loss Distribution in the Pixel Sensor

Within the detector material, ionization and its simultaneous energy transfer caused by the traversing charged particles is a random process, characterized by the quantum mechanical behavior of the interacting particles. Therefore, the energy loss of an ionizing incident particle cannot be deterministic, same as most phenomena in particle physics, but randomly arises around a most probably value (MPV) following a certain type of probability distribution.

The Bethe-Bloch formula delivers the mean energy loss $\left\langle \frac{dE}{dx} \right\rangle$ of the primary particles traversing the material, as mentioned in 3.1.1. Occasionally, the traversing particle transfers extraordinary amount of energy to the atomic electrons during ionizing collisions, hence bearing an energy loss far more greater than the mean value while producing
very energetic electrons. Such fast electrons would then collide with other atomic electrons in material and cause further ionization. In the bubble chamber, as shown in Figure 3.5, they are commonly observed as the “hairy” trails along the main tracks and by definition referred as the $\delta$ electrons or $\delta$ ray.

![Figure 3.5: $\delta$ electrons are observed in the CERN 2-metre hydrogen bubble chamber. The $K^-$ beam particles left the parallel trails through the ionization of hydrogen, while the $\delta$ electrons are easily recognizable indicated by their tracks spiral. Adapted from [19].](image)

Since the free charges created by the ionizing delta-electrons would also be collected at the electrodes and constitute the signal currents, the “unusually” large energy loss can thus be measured with certain level of precision. For many types of materials, including silicon which is the reaction material of the PXD sensors, the energy loss distribution established from the charge measurements can be approximately assigned to the Landau distribution. A typical normalized Landau distribution, as illustrated in Figure 3.6, has a long tail towards the positive infinity, which naturally corresponds to the rarely occurring high-energy-transfer collisions.

As a result of the long tails, the moments (mean, variance) of the Landau distribution are undefined, thus the distribution is parametrized with the peak value (MPV) and a scale factor. For thick absorbers, it appears that the tails of the energy loss distribution tend to be severely contained, which indicates that an upper bound on the energy transfer shall exist and hence, the actual energy loss distribution should be regarded as a quasi Landau distribution, although this effect remains hardly noticeable for thin absorbers.

In the coming section, the Landau distribution will be integrated into program for simulating the energy deposition of the Monte Carlo particles within each individual pixel sensor.
3.2 The PXD Toy Simulation

For the development of the local reconstruction algorithms associated with the Pixel Detector, the Toy Simulation is implemented. Unlike the full PXD simulation brought by the PXD module of basf2, this simulation only sets up a small area of pixel sensors while disregarding many physics conditions that are concerned in the full simulation. For example, the magnetic field is absent, as the simulated particles are simply taken to be straight track lines, without any physics properties such as charge or momentum taken into account. It is believed to be fully sufficient, that such a small-scaled, simplified simulation frame is able to fulfill the purpose of developing and testing the PXD local reconstruction algorithms.

3.2.1 General Structure

In the Toy Simulation, primarily a pixel matrix illustrating a small area of the PXD wafer is built. To do so, a pixel layout is integrated with 100 cubical volumes aligned \(10 \times 10\) in two dimensions. Each cubical volume represents one pixel sensor of PXD and is given a side length of one single unit, numerically 1. This area, as shown in Figure 3.7, serves as the unique interaction environment for the Toy Simulation. The reason of choosing the size of “\(10 \times 10\)” for the pixel matrix, is that the simulation was designed for studying only single track events, which in reality mostly produce rather small pixel clusters. Thus, the chosen size is supposed to be sufficient for the simulation task.
Furthermore, to simulate the traversing particles, straight track lines can be generated either with the position and direction manually selected or purely randomly. A random track has to be created within the area of the two-dimensional pixel array. Naturally it represents one ionizing particle intersecting the pixel layer with no specific physics property integrated. The position of each track line is restricted, so that its intersecting point with the pixel layer’s bottom plane only settles on the bottom facet of a pixel cube. Hence the bottom plane is considered to be where the incident particles enter the wafer, as in the real geometry configuration. The direction of each track line can be set as uniformly randomized, thus the rare cases of small angle entries which lead to larger clusters would also take place. This is proven to be rather harmless to the testing and development of the local reconstruction algorithms and was therefore adapted.

Finally, through adding the physics effects and the energy cut-off threshold, the track length of any particle line within each pixel volume is first calculated and then converted to the corresponding energy deposition. Thus the expected outcome of a run of Toy Simulation, namely an energy loss cluster with a pattern close to a measured PXD cluster, is obtained and ready for further applications.
3.2.2 Physics Effects

The physics effects just mentioned above are already partially explained in section 3.1. They are the Gaussian-distributed electronic background and the Landau-distributed specific energy loss $\frac{dE}{dx}$, respectively. In addition, a cut-off threshold on the simulated pixels’ charge will also be applied after the physics effects being taken into account.

The background signals in the PXD sensors are mainly caused by the leakage current which was explained in the last section. In the depleted bulk there is still a certain amount of free charge carriers, especially when the cooling was insufficient among the pixels sensors. When the pixels are live, hence when the detector is turned on, these free charges would drift to the electrodes and consequently generate the noise currents. Moreover, the read-out process also contributes to the noise signals. In the Toy Simulation, the background noise within each pixel sensor is set to be Gaussian distributed, with the mean set as 0 and the variance as 0.03.

As introduced in section 3.1, the ionization process takes place with certain low-probability ionizing collisions accompanying high energy transfers while creating $\delta$ electrons, and hence lead to the long tails in the energy loss distribution. The Landau randomized specific energy loss is naturally taken to generate the energy deposit of the traversing particles within each pixel cube. For every traversed pixel, the specific energy loss ($\frac{dE}{dx}$) is randomly generated according to Landau distribution with the moments: \( MPV = 7, \text{width} = 0.8 \). These parameters are acquired by scaling the data adapted from Figure 3.8. In this figure, the momentum dependence of the Landau parameters for electrons intersecting the PXD was fitted. An approximate adaption with the MPV taken as $700 \times 10^3$ and the width as $80 \times 10^3$ was made. For more clarity, these values are scaled to the above mentioned ones after being divided by $10^5$. At the end, through multiplying the Landau-distributed $\frac{dE}{dx}$ and the calculated track length within any traversed pixel, the energy deposit of that pixel cube will then be obtained.

For a run of the Toy Simulation, after generating the pixel sensors’ background data and the energy deposition of a single random track for the $10 \times 10$ pixel layer, a two dimensional array representing the measured pixel charges is simply created by summing the non-zero energy loss and the Gaussian background charge for each pixel cube. To subtract the valid data cluster for analysis, hence excluding the pixels that only carry the background charges and are not actually traversed by the particle track, a cut-off threshold shall be applied. It allows only pixels possessing charge values above this threshold to be counted as the traversed ones, which would form a simulated PXD cluster for analyzing the intersecting position of the Monte Carlo particle. The threshold value
in the Toy Simulation is chosen to be three times of the background variance [21], hence

\[ \text{Threshold} = 3 \times 0.03 \times 7, \]

as 0.63, respectively. It should be mentioned, that since the parameters of the Gaussian-distributed background are estimated regarding the side length of the pixel cubes, therefore the applicable background noise values will always be the product of the randomly generated number and the MPV of the adapted \( \frac{dE}{dx} \) distribution, namely 7.

### 3.2.3 Technical Procedure

The Toy Simulation is a self-confined simulation program independent of basf2. It consists of several classes which set up the simulation frame and realize the above mentioned physics effects. In the main program – “ToySim.cc”, these classes can be applied to achieve the Toy Simulation for arbitrary number of single track events. Here the classes will be briefly introduced with an overview on their structure and general functionality.

- **The Facet Class**: To form a pixel cube, six classes representing 6 facets of any particular pixel are first integrated. They are the classes “\texttt{Facet}_n.h”, with integer “n” varies from 1 to 6, each denoting one specific facet of a standard cubical volume. Each “facet class” object can be related to any pixel with a certain set of position indices, \((u, v)\), respectively. Thus the facet object not only contains information such as the direction vector of the facet’s plane, but also the boundaries identifying the facet’s own area on its plane, which are the coordinates intervals for three dimensions that are characterized by the pixel’s the position indices \((u, v)\) and the **facet integer** “n”. The information confined within such an object can therefore be conveniently adapted for searching and locating the intersection point between a single particle track and a pixel’s facet.
• **The Pixel Class**: By including the six facet classes, the pixel class - “Pixel.h” can be used to construct the facet objects which correspond to a specific pixel. A pixel class object also stores the pixel’s position indices and its “energy deposition”, which is essentially the length of the track segment within the pixel cube. Provided a pixel’s index \((u, v)\), the geometrical and physical information of any pixel from the 10 \times 10 two-dimensional sensor array can be easily accessed using the corresponding class object.

• **The Track Class**: The class – “Track.h” is made to create particle track objects which store a track line’s direction and position. Additionally, the methods integrated in this class can be applied to calculate the intersection point of the track object with any plane in space, and most importantly, to acquire the “energy loss” of a particle track within any pixel with the related pixel object simply taken as the parameter for the corresponding method. During one simulation, the track line object representing one particle is first generated and this method would be launched repeatedly while taking each of the 100 pixels as its parameter, hence produces a cluster with non-zero track lengths being calculated. This *track lengths cluster* will then be converted to the actual energy loss cluster with the physics effects taken into account.

• **The Physics Effects Class**: The “PhysEffect.h” adapts the ROOT class “TRandom1.h” for generating random numbers under Gaussian and Landau distribution. The key method of this class takes the 10 \times 10 track lengths array as its parameter and returns the energy loss cluster concerning all the physics effects analyzed in subsection 3.2.2. This method assigns the Gaussian distributed background charge to all the array elements while converting the track lengths to the “measured charge” with the specific energy loss randomly generated under the Landau distribution. At the end the cut-off threshold is applied on all the array elements to leave the “background” elements filtered.

### 3.2.4 Graphical Illustration

For a straight view on the effects brought by the Toy Simulation, the charge deposition in the 10 \times 10 pixel array that arises after a run of the simulation program is visualized in the following plots. Figure 3.9 and 3.10 demonstrate a particle intersecting the pixel layer and leaves the lengths of the track segments as the “charge deposit”. Figure 3.11 which is a lego histogram generated by ROOT provides a graphical comparison of the track lengths and their corresponding energy deposit resulted by a single track event in the Toy Simulation. The fill-in values for the lengths of track segments are scaled by multiplying the MPV of the adapted energy loss distribution, for the purpose of a clearer view.
Figure 3.9: A single track event illustrated. The color depth of the traversed pixels displays the difference between the lengths of track segments inside the pixels.

Figure 3.10: Top view of a single track event.
Figure 3.11: A lego plot illustrating the scaled track lengths (red) and their converted energy deposit (blue) generated by a run of Toy Simulation.
Chapter 4

Local Reconstruction Algorithms for the Pixel Detector

As noted in Chapter 1, the Belle II Experiment is dedicated to look for the New Physics phenomena and deepen our present understanding of CP-violation. To achieve this, the positions of the $B/\bar{B}$, $D/\bar{D}$ decay vertices must be precisely measured. Therefore, the Belle II Vertex Detector, composed of the Silicon Vertex Detector (SVD) and the Pixel Detector (PXD), is set to provide accurate tracking parameters of the charged particles created by the $B/D$-decay, which is of key interest for vertexing.

The Pixel Detector (PXD), as introduced in Chapter 2, is designed to be capable of delivering a superb performance in measuring the intersecting positions of the incident particles. In principle, this should be achieved on the perspectives of both technical engineering and data analysis. For the latter one, the development of algorithms which process the cluster data collected by the PXD is of particular interest. The local reconstruction algorithms are expected to make precise estimations for the incident particles’ track position through analyzing the measured pixel cluster data. In this chapter, the present Center of Gravity (COG) algorithm integrated into the PXD module of basf2 will first be looked into, with its performance on the PXD local reconstruction closely examined. Following this, a thorough study on the new “Shift algorithm” will be given. By far this algorithm is believed to be able to provide a considerably sharper in-plane-resolution than its predecessor. To demonstrate this, it will be comprehensively tested with the simulation data produced by Toy Simulation. At the same time the testing results will be analyzed in comparison with the ones generated from applying the Center of Gravity algorithm on the same sets of simulation data.

Since it has been carefully proven, that a higher in-plane-resolution does lead to a better vertexing result, the new algorithm will also be briefly examined to explain the
ways of its potential implementation into the current PXD Module of basf2, hence to be applied in a full-scaled PXD simulation.

4.1 Local Hit Reconstruction with the Center of Gravity Algorithm

The Center of Gravity (COG) algorithm is one of the most commonly used local reconstruction algorithms for estimating the hit position on the silicon vertex detectors. In the classical Turchetta’s paper [23], the COG algorithm was introduced as a position-finding-algorithm (PFA) for one-dimensional position finding. Back then only the silicon microstrip detectors were in use as the vertex detector, while the pixel detectors not yet invented, the clusters of strips activated by the incident particles have to be first located by applying the Cluster Finding Algorithms (CFA), then the appropriate position-finding-algorithm will be chosen to determine the approximate hit positions. For the one-dimensional case mentioned earlier, the hit position can be estimated according to the COG algorithm, once the strip cluster is identified:

\[ X_{COG} = \frac{\sum_{\text{cluster}} (S_i \cdot x_i)}{\sum_{\text{cluster}} S_i}, \]

Here \(x_i\) is the position of the \(i\)th strip included in the cluster, and \(S_i\) stands for the charge signal collected on that strip. The sum of the charge signals of all cluster strips corresponds to the denominator. This weighted average on the positions of the bypassed strips provides the rough hit position of an incident particle while the “mass” taken to be the charge collected on each strip, thus the algorithm is referred as the “Center of Gravity” algorithm.

For the new generation of experiments at high energy colliders, *pixel detectors* are widely adapted for their outstanding tracking performance. With the most advanced technological integration, the Belle II PXD is able to provide a significantly better resolution on finding the hit positions than the previous strip detectors. Still, for an idealized functioning of a pixel detector, not only its hardware design and engineering should be constantly concerned with emphasis, but also the local reconstruction algorithm which processes the measured cluster data should be overly enhanced or updated to achieve an ideally minimized vertexing error.

At the beginning of this “pixel era”, the Center of Gravity algorithm is naturally inherited as the default local reconstruction algorithm for the pixel detector. For the Belle II PXD simulation, this algorithm is currently integrated into basf2 and being
applied to find the hit position of each valid pixel cluster. Similar to the principle applied
on the silicon strip detector, the COG algorithm still takes the collected charge signal
of each pixel sensor as the “mass” and sets the weighted average on the positions of the
involved cluster pixels as the estimated hit position. When applied two-dimensionally,
the Center of Gravity algorithm for the pixel detector can be illustrated as follows:

\[
x_{\text{COG}} = \frac{\sum_{u,v} (C_{uv} \cdot x_{uv})}{\sum_{u,v} C_{uv}},
\]

\[
y_{\text{COG}} = \frac{\sum_{u,v} (C_{uv} \cdot y_{uv})}{\sum_{u,v} C_{uv}}.
\]

In the equations above, \( C_{uv} \) denotes the charge signal measured in the pixel sensor
which is referred to with the two-dimensional indices \( u \) and \( v \). As for the Toy Simulation
introduced in Chapter 3, the “\( u \)” index varies from 0 to 9 and represents the pixel index in
the x-dimension, as the “\( v \)” index varies in the same range denoting the pixel index in the
y-dimension. Thus \( x_{uv} \) and \( y_{uv} \) characterize the position coordinates of the pixel \((u, v)\)
in the x-y plane, respectively. In the Belle II PXD Toy Simulation, each pixel sensor is
set up as a cubical volume with a side length of unit 1, therefore \((x_{uv}, y_{uv})\) is chosen to be
the geometrical center of the \((u, v)\) pixel’s facet which is parallel to the x-y plane. As a
result, the estimated hit position \((x_{\text{COG}}, y_{\text{COG}})\) is actually only an approximation of the
actual intersecting point of the incident particle’s track line with the so-called “middle
plane” of the pixel layer. Here the lengths of an incident particle’s track segments within
the traversed pixels are directly taken as the “charge signals”, with no concerns of the
Gaussian distributed background noise or the Landau randomized charge distribution.
It should be emphasized, that due to the “mathematical imperfection” of the COG
algorithm, even for this scenario where the lengths of the track segments are taken as
the “mass” of the pixels, the outcome of this algorithm still presents quite noticeable
deviation with respect to the actual hit position. Only for the special case shown in
Figure 4.1, where the particle’s trajectory has an angle of 30 degree to the x-y plane
while its projection on the x-y plane has an angle of 45 degree to the x/y axis, the COG
algorithm could deliver an exact agreement with the real hit position on the “middle
plane” of the pixel layer.

When the physics effects discussed in the last chapter are considered, hence when the
Center of Gravity algorithm is applied on the pixel cluster data generated by a Toy
Simulation, the estimated hit positions are expected to have even lower resolution than
the previous “idealized” case. In order to compare the effects brought by the COG al-
gorithm on these two cases, the Toy Simulation is executed 1000 times, thus producing
1000 sets of cluster data for the COG algorithm to operate on, the resulting x and y
deviations of the estimated hit positions are represented in Figure 4.2. Similarly, the
Figure 4.1: The only track orientation with which the COG algorithm works perfectly. Blue arrow stands for the particle's track direction. Orange arrow marks its projection on the x-y plane. The dashed line on the x-y plane is parallel to the x-axis.

x and y deviations of the estimated intersecting positions produced by the Center of Gravity algorithm while only using the track lengths as the “mass”, which are also obtained from 1000 randomly generated tracks simulating the traversing incident particles, are illustrated in Figure 4.3. Clearly, applied with the COG algorithm, the hit position resolution for the case of Toy Simulation clusters is much lower than the one with track length clusters.

Clearly the Center of Gravity algorithm leaves certain space for enhancements on the PXD local reconstruction, yet it still provides fairly acceptable track position parameters and is currently applied in the simulation tasks of basf2. Additionally, this algorithm does not require the information such as incident particles' direction of propagation or their rough entry position on the PXD wafers, therefore the COG algorithm is regarded as a relatively fast and convenient way of locating an approximate hit position through the simple processing of the pixel cluster data. In addition, when the pixel clusters were hard to be arranged to the particle tracks under certain circumstances, the Center of Gravity algorithm can also be used to link a target cluster to its corresponding track that is reconstructed by the CDC and the SVD. This important function that the COG algorithm provides is of particular interest for the new PXD local reconstruction algorithm that will be introduced in the next section.
Figure 4.2: Position error distribution for 1000 Toy Simulation clusters processed by the COG algorithm.

Figure 4.3: Position error distribution for 1000 track length clusters processed by the COG algorithm.
4.2 The Shift Algorithm

Though the classical Center of Gravity algorithm is capable of delivering an acceptable in-plane-resolution, while in pursuit of the enhanced local reconstruction performance of the Belle II pixel detector, a different approach for estimating the PXD hit position will be examined in this section, with analysis made on its general performance in comparison with the currently integrated COG algorithm.

4.2.1 Motivation and Principles

As reflected in section 4.1, the Center of Gravity algorithm is proven to be non-optimal for locating the true hit position when the lengths of the track segments within each pixel cube were simply taken as the charge deposit. For some cases, depending on the direction of the particle track, the deviation between the estimated hit position and the true intersecting point of the incident particle could appear quite significant. In order to enhance the local reconstruction resolution for this “idealized” scenario, the Shift algorithm is proposed.

Compared to the Center of Gravity algorithm, the Shift algorithm is a rather “mechanical” method of locating the intersecting position of the traversing particle’s trajectory with the pixel layer’s “middle plane”. Assuming the incident particle’s track direction and its approximate entry position on the pixel layer are given, the algorithm practically generates multiple track lines which have the exact same angular direction as the incident particle. These tracks should be created within a certain area, usually their intersecting positions with the “bottom plane” of the pixel layer should be bounded within the pixel’s bottom facet which is the first pixel sensor that is traversed by the incident particle, hence the facet where the track line actually enters the pixel layer.

An illustration of the Shift algorithm is shown in Figure 4.4, 4.5. For the facet “A”, which is the “entering facet” of the incident particle, test tracks parallel to the particle’s track line are generated one by one, two-dimensionally, from the facet’s “upper left corner” to its “lower right corner”. Hence, while the direction of the “shifted” test track remains constant, its intersecting position with the entering facet “A” is shifted from left to right, row after row within this facet, with a selected shift spread of 0.01 for both columns and rows, which is only 1 percent of the unit side length of the pixel cube. Therefore, 100^2 such intersecting points representing 10000 shifted test tracks are extracted for calculation. Each test track can be mathematically identified with the common track direction vector and the two-dimensional coordinates of its intersecting position with the facet. Furthermore, the track segments’ lengths of each shifted test
track within the pixels which are the ones only traversed by the original incident particle
are calculated inside the frame of the Toy Simulation. These “charge signals” produced
by any shifted test track will be subtracted by the “true charge deposit” separately with
respect to each corresponding pixel, then the absolute values of these differences will be
summed up and the resulting overall charge deviation produced between a shifted test
track and the incident particle will be stored for the purpose of comparison. Naturally,
among all the 10000 shifted test tracks, the one which possesses the minimal overall
charge deviation will be selected to become the estimated trajectory for the traversing
incident particle. Its intersecting point with the pixel layer’s middle plane will then be
taken as the estimated hit position.

\[ \text{Track}_{\text{estimated}} = \text{Track}_{\text{min}} \left( \sum_{\text{cluster}} |C_{\text{test},i} - C_{\text{incident},i}| \right), \]

**Figure 4.4:** A small area of the entrance facet is illustrated. With the test tracks’
direction given, the shifting positions vary two-dimensionally along the x and y direction
with a spread of 0.01, as marked by the black dots. The blue dot stands for an incident
particle’s intersecting position, while the estimated track’s position denoted by the red
dot.

Furthermore, the mathematical formulation of the Shift algorithm is given as:
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Figure 4.5: An illustration of the shifted tracks. The blue one denotes the true trajectory of the incident particle, the green one for the estimated track’s position that is at the closest range to the real track line, with respect to the red ones that are further away and hence excluded from being taken as the estimated track.

where $C_{\text{test},i}$ and $C_{\text{incident},i}$ refer to the “charge deposit” caused by a test track and the incident particle within the $i$th pixel of the cluster. Here for the idealized case, they are just the lengths of the track segments within the $i$th pixel of the corresponding test and particle tracks.

Since the shift spread is only set equal to one percent of the facet’s unit length, for the pixel clusters that have a two-dimensional pixel-layout, the Shift algorithm can therefore deliver an optimal resolution of less than 0.01 in both $x$ and $y$ dimension. But for a cluster containing only one pixel or all its active pixels were aligned one-dimensionally, the Shift algorithm obviously cannot provide the same precision. For the case of 1-pixel cluster the shift-and-compare process is in theory rather meaningless and could lead to quite large error. Consequently, for this case the Shift algorithm will be abandoned and it will again follow the Center of Gravity algorithm by simply taking the geometrical center of the pixel cube as the estimated hit position. For the other case where the cluster is made of a one-dimensional pixel-array, either in $x$- or $y$-dimension, as demonstrated in Figure 4.6, the hit position resolution can still remain excellent for the dimension in which the pixels are aligned to, but the resolution will be much lower for the estimated
hit coordinate in the orthogonal dimension. Reason of this is due to the two-dimensional shifting process, similar to the 1-pixel cluster case, the algorithm does not distinguish among the multiple test tracks that have the same overall charge deviation which is indeed minimal, and hence is unable to identify the test track that is truly at the closest range to the incident particle’s track line.

Compared to the Center of Gravity algorithm, the Shift algorithm does lead to an overall enhanced hit position resolution, as shown in Figure 4.7, for 1000 randomly generated tracks representing 1000 incident particles, in the idealized scenario where the track lengths are taken to be the charge signals, the improvement made on the tracking precision is indeed significant, though many one-pixel clusters and clusters with one-dimensional pixel-layout were not excluded. When tested with the data generated by 1000 tracks which only traverse the pixel cubes in a two-dimensional alignment, the Shift algorithm apparently provides an almost perfect position finding performance, as shown in Figure 4.8.

It should be again noticed, that the Shift algorithm requires the direction and the rough entry position of the incident particle that leads to a pixel cluster, although such crucial information is not needed by the Center of Gravity algorithm and may be difficult to obtain for some cases. Therefore the Shift algorithm could on occasion be applied as an extension rather than a complete replacement of the COG algorithm. For instance, when the corresponding particle’s track line could not be determined for one PXD cluster, the hit position estimated by the Center of Gravity algorithm can be adapted to select and extrapolate the correct reconstructed track line from the tracking detector back to the pixel cluster, thus providing the essential directional information of the incident particle and accordingly the Shift algorithm will then be ready for application.
Figure 4.7: Position error distribution for 1000 track length clusters processed by the Shift (green) and the COG (red) algorithm.
Figure 4.8: Position error distribution for 1000 track length clusters in 2D alignment processed by the Shift (green) and the COG (red) algorithm.
4.2.2 Testing with the Toy Simulation

In the previous subsection, in order to demonstrate the principles and the basic properties of the new position finding algorithm, the charge deposit of each traversed pixel was simply taken to be the length of the track segment inside each cubical pixel volume. This rough approximation was also applied during the initial development of the Shift algorithm. To gain a view on its performance in a relatively more realistic scenario, the algorithm will be exercised with the cluster data generated by the Toy Simulation.

As explained in the last chapter, the Toy Simulation randomly converts the track length within one pixel to a value representing the charge deposit according to the Landau distribution. In addition, it adds approximated effects of the Gaussian distributed electronic background with a cut-off value taken into account. The pixel cluster data generated this way is considered as a close approximation to the PXD clusters gathered through experimental measurements. Hence the hit position resolution led by the Shift algorithm while applied with the Toy Simulation data should make a convincing point that the new algorithm is able to outperform its predecessor on a "quasi-realistic" level.

As shown in Figure 4.9, tested with 1000 sets of Toy Simulation clusters, the Shift algorithm clearly still provides a leading hit position resolution in both x and y dimensions compared to the COG algorithm. Since the two-dimensional PXD clusters are usually created in a dominant amount compared to the one-dimensional or single-pixel clusters, it is worth of demonstrating the performance of the Shift algorithm handling only the 2D-clusters, which is illustrated in Figure 4.10, 4.11. It should be noticed, that the RMS values of the error distributions are strongly related to the widths of the distribution histograms. Since the values under various conditions are distributed quasi-symmetric around 0, hence smaller RMS corresponds to a better hit position resolution, as reflected in the histograms.

In comparison with the “idealized” scenario, the hit position resolution resulted by applying the Shift algorithm is naturally reduced mainly due to the randomized conversion that was operated on the track lengths. The “shift-and-compare” process remains exactly the same as described in the previous subsection and the algorithm should still be formulated as:

$$\text{Track}_{\text{estimated}} = \text{Track}_{\text{min}} \left( \sum_{\text{cluster}} |C_{\text{test},i} - C_{\text{incident},i}| \right),$$

where $C_{\text{incident},i}$ denotes the charge deposit generated by the Toy Simulation.
Figure 4.9: Position error distribution for 1000 Toy Simulation clusters processed by the Shift (green) and the COG (red) algorithm. $\text{RMS}_{\text{Shift}} = 0.2314$, $\text{RMS}_{\text{COG}} = 0.2385$. $\text{RMS}_{\text{Shift}} = 0.2241$, $\text{RMS}_{\text{COG}} = 0.2372$. 
Figure 4.10: Position error distribution for 1000 Toy Simulation clusters in 2D alignment processed by the Shift (green) and the COG (red) algorithm. $RMS_{\text{Shift}-x} = 0.1193$, $RMS_{\text{COG}-x} = 0.2477$, $RMS_{\text{Shift}-y} = 0.1168$, $RMS_{\text{COG}-y} = 0.2466$. 
It ought to be mentioned, that $C_{\text{test},i}$ is chosen to be the product of the actual track length and the most probable value (MPV) of the Landau distribution used in the Toy Simulation. In this way, the overall hit position resolution was examined of being optimized with respect to the usage of other values for generating the “charge deposit” of the shifted test tracks. Consequently, it is conceivable that the same method of setting the $C_{\text{test},i}$ value should be used when the Shift algorithm is applied with other forms of PXD data. For the PXD clusters that are generated from the experimental measurements, the most probable value of the specific energy loss distribution could be obtained through fitting the observed data sets of $\frac{dE}{dx}$.

### 4.2.3 The Algorithm Efficiency

The preliminary testing with the Toy Simulation data clearly demonstrates the advantage of the Shift algorithm for estimating the hit position within a $10 \times 10$ pixel alignment. But the rather mechanical “shift-and-compare” process could be considered as potentially *inefficient* and hence would have led to an unacceptably low speed once it is integrated into basf2 for data analysis.
During the initial development of the algorithm, when analyzing a given PXD cluster, each of the 10000 shifted test tracks was brought into calculation for obtaining its track lengths within all the 100 pixel cubes. This procedure was literally adapted from the Toy Simulation and is proven to be inappropriate for the purpose of position finding. For instance, the time consumed for analyzing 1000 Toy Simulation clusters was approximately 134 seconds, including the time of executing 1000 Toy Simulations which can actually be neglected. Obviously, this kind of inefficiency cannot be accepted and the algorithm was therefore modified, so that the shifted test tracks are set to check with the pixels that only belong to a PXD cluster while all the other pixels in the array being left out of the calculation. By doing this, the time of processing 1000 clusters produced by the Toy Simulation plus the time cost by the simulations was drastically reduced to about 8 seconds. With averagely less than 0.01 seconds required to analyze one Toy Simulation cluster, the Shift algorithm can be regarded as being fully competent to provide a desirable efficiency while processing large-scaled PXD data sets.

4.3 Further Testing and Integration

4.3.1 Testing with basf2 Simulation Data

As mentioned in the previous section, the Toy Simulation was designed to provide a testing environment on a “quasi-realistic” level. For a full confirmation that the Shift algorithm can also deliver an outstanding performance while handling the experimental data, it is natural to apply the PXD cluster data generated by basf2 simulations for further testing of the algorithm. The PXD Module and the other associating modules of basf2 are able to simulate various physics effects that are not concerned in the Toy Simulation, hence it can assure a higher level of likeness with respect to the experiments’ scenario within which the “real” PXD clusters are generated.

Compared to the Toy Simulation, basf2 has a much lower degree of approximation in creating the Landau randomized charge deposit and the electronic background noise. In addition, it takes account of various physics conditions, e.g. the Lorentz Effect brought by the magnetic field dominating the area where the PXD is located. Therefore, the test performance of the Shift algorithm within the frame of basf2 shall give an undoubted view on its superiority of being an advanced position finding algorithm for the pixel detectors.

However, due to unexpected difficulties and the time limit, the above mentioned testing will not be studied within the scope of this thesis. The problem was discovered after running one PXD simulation in basf2. It was noticed, that the geometrical structure of
the current Toy Simulation pixels deviates from the one within the basf2 frame. The cluster data that was produced by the basf2 simulation reveals, that the geometrical form of a DEPFET sensor used in Belle II experiment, as well as in the basf2 PXD Module, is actually of cuboid, rather than the cubical shape adapted in the Toy Simulation. As shown in Figure 4.12, the thickness (“height”) of a pixel sensor is 75 microns, the side length in one direction is of 50 microns, respectively, as in the other direction the side length varies from 55 to 85 microns, depending on the channel for which a pixel sensor is designed.

Consequently, this geometrical discrepancy would lead to quite severe inaccuracy if the cluster data generated by a basf2 simulation was directly implemented into the current $10 \times 10$ pixel layer in which the Toy Simulation and the Shift algorithm are practiced. Apparently, if one basf2 cluster was adapted into the cube-shaped pixels’ alignment, the Monte Carlo particle’s track position can hardly remain true and the energy deposit of each shifted test track would have much less relevance with respect to the cluster values. Therefore, the idea of testing the Shift algorithm on the basf2 clusters through adapting the approximated Toy Simulation geometry has been excluded.

Since the current Center of Gravity algorithm is not affected by the geometrical variety of the pixel sensors, to truly make use of the Shift algorithm within the frame of
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4.3.2 Integration into basf2

Once the performance of the Shift algorithm was confirmed in the highly realistic scenario that was described in the last subsection, it will be worthy for the algorithm being integrated into basf2, hence to replace the current Center of Gravity algorithm for the purpose of precise PXD local reconstruction. To fully realize this integration, despite of the previously noted classes, one extra class that stores the directional information of the incident particle’s track line and eventually identifies the particle’s entry facet will be needed.

It ought to be mentioned, there is analysis that has been made [24] which gave rise to the conclusion, that a higher in-plane-resolution on the PXD layers would indeed lead to a sharpened vertexing resolution. Therefore it is expected, that after integrating the Shift algorithm into basf2, the spatial error of the reconstructed vertices would noticeably decrease. And this effect can accordingly be observed by using the Display
Module of basf2 through the visualization and comparison of the error ellipsoids which represent the covariance matrices (spatial error) of the reconstructed vertices.

It is anticipated, that the size of this error ellipsoid would be significantly diminished after the Shift algorithm being successfully integrated into basf2, serving as the main local reconstruction algorithm for the Pixel Detector.
Chapter 5

Conclusions

In Chapter 3 the PXD Toy Simulation was developed. This simulation frame sets up a relatively small area of cubical-shaped pixel sensors, concerning only 100 pixels aligned two-dimensionally in a $10 \times 10$ array. In addition, limited physics effects as the Gaussian-distributed background noise and the Landau-distributed specific energy loss are simulated with certain level of simplification, with as well a cut-off threshold value taken into account. The Toy Simulation serves to provide a reliable and efficient simulation environment for the testing and development of the Belle II PXD local reconstruction algorithms.

In Chapter 4 a new type of position-finding-algorithm for pixel detectors was created and tested in comparison with the currently implemented Center of Gravity algorithm. This algorithm is proven to be able to strongly outperform the COG algorithm under various circumstances with an acceptable efficiency that wouldn’t cause severe slowdown for the general data processing of Bell II. It is expected, that the Shift algorithm would remain stable when it’s tested with the cluster data generated by the basf2 PXD simulation which contains a slightly different pixel geometry and a more comprehensive physics environment.

Overall, as a local position reconstruction algorithm for the innermost vertex detector of Belle II, and potentially also for similar vertex detectors of other upcoming High Energy Collider experiments, the Shift algorithm is believed to be capable of replacing or at least extending the currently applied Center of Gravity algorithm to achieve a considerably improved vertexing performance, and hence to bring a fraction of advantage into the study of New Physics.
Bibliography


[10] Belle Collaboration I. Adachi et al. Precise measurement of the cp violation parameter $sin2\phi_1$ in $b^0 \rightarrow (c\bar{c})k^0$ decays. Physical Review Letters, 108(17), April 2012.


